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Using Neural Networks in the Mapping of Mixed Discrete/Continuous Design Spaces With Application to Structural Design

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The objective of this task was to extend recent research efforts in order to evaluate the suitability of using artificial neural networks to provide quantitative design space representations for structural concepts with combined continuous and discrete design variables.

For a simple structural problem containing both discrete and continuous design variables it was demonstrated that the character of the design space could be well represented with a relatively small amount of training data. It was also shown that design methods are available which can be used for mixed discrete/continuous design variable problems. These methods are however limited by the discontinuous nature of the discrete design problem and by the ability to predict system characteristics in an efficient manner. Using an approach with feed-forward, back-propagation neural networks an efficient method for the design of mixed discrete/continuous systems can be obtained.

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1 INTRODUCTION

The design of aircraft subsystems typically requires large amounts of information which must be evaluated in order to obtain improved subsystem configurations. Traditional optimization techniques can be used to find the solution to the design problem when the system in question is composed solely of either continuous (such as wing span) or discrete (material properties, number of spars...) design variables. These techniques, however, are in many cases inadequate when faced with the task of optimizing mixed discrete/continuous systems.

Considering the vast amount of information required for the aircraft system design problem, how do we generate "optimal" designs with constraints on time and computational resources? Further complicating the subsystem design problem is the fact that discrete design variable optimization generally requires a procedure such as simulated annealing or a genetic algorithm which require an extremely large number of objective function evaluations. The reduction of the number of these objective function evaluations is a high priority. The purpose of this study was to address issues pertaining to design space representation and evaluation for structural design problems which contain both discrete and continuous design variables.

The method by which the design space definition for the mixed discrete/continuous design variable problem was addressed in this study was through the use of artificial neural networks. These networks have been shown to provide a useful tool for storage and manipulation of design data obtained from conventional analysis techniques for systems of either continuous[1-3] or discrete[1,4-6] design variables, but realistic design problems containing both continuous and discrete variables have not yet been considered in detail[†]. The current research employed feed-forward, back propagation neural networks to provide an approximation to a mixed discrete/continuous design space.

A hierarchic design problem was considered in which conventional numerical structural analysis and optimization methods were used to develop the design information at the lowest level of the hierarchy. These methods provided "feasible designs" in which only a part or "subspace" of the complete set of system design variables were considered. The information determined for these feasible designs was then used to "train"

† All references are cited in the text as superscripts and appear listed in sequence at the end of the report.

the neural networks. The resulting neural networks were used to perform an optimization using the remaining design variables which were both discrete and continuous in nature.

The methods considered in this study were evaluated by application to a relatively straightforward structural design problem. This problem was selected since it provided the basic characteristics of both continuous and discrete design variables and was simple enough to allow for graphical presentation of various results. The sections which follow briefly describe the specific design problem, the way in which neural networks were used to map the design space for this problem and the methodology used to obtain optimal solutions by using the neural networks. This is followed by a more detailed discussion of results when these methods were applied to the simple structural example problem.

2 THE DESIGN PROBLEM

The problem under investigation in this study was the conceptual design of a structure for which the design vector was comprised of both discrete and continuous design variables. Since the focus of the investigation was the mapping of design spaces containing both discrete and continuous design variables it was desired that the problem to which the design space mapping techniques were applied be easily analyzed, yet provide a design space with realistic complexity.

The "design problem" as posed for this study was actually a sequence of two problems, or a two-step heirachic design problem, with unique design variables at each level and the same merit function. The first step was a subspace optimization using continuous design variables only and a design algorithm based upon an optimality criteria. The second step was the continuous/discrete design problem which used the neural network. When one considers the design of a structure it typically involves the selection of the basic geometric arrangement, or configuration, the selection of the materials to be used in the structure, the material system, and finally the "sizing" of the components which make up the structure. There have been many methods developed which allow for the sizing or "optimization" of structural components for a given configuration and material properties and these are often based upon finite element representations of the structure. The selection of the component sizes for a given structure is referred to as a "subspace" optimization.

Including the selection of the material system and configuration in the design "optimization" may allow for even better designs but these types of design variables are more difficult to include in many of the current optimization schemes. This is often the result of the fact that the finite element method used to model and analyze the structure is not as easily adaptable to variations in configuration and material properties as it is to variations in component sizing. An additional complexity is due to the the discrete nature of some of the configuration and material property design variables and the difficulty associated with using either optimality criteria or "gradient" based optimization algorithms to perform the design process.

In the current study, traditional finite element analysis techniques were used to model and analyze the structure. For a given configuration and material properties a simple optimality criteria design procedure was used to determine the "best" design for that configuration and material system. Then the design space represented by the configuration and materials design variables was evaluated to determine the configuration and materials which yielded the "best" of the "best" designs. In order to provide graphical representations of the design space for this simple problem, the candidate structure was selected so that the number of discrete design variables was kept small. It was decided that the design of a five bar truss structure could provide a realistically complex design space while the subspace optimization could be accomplished quickly and without investing large amounts of time in developing an extensive set of analysis and optimization software. Described below is the simple structural example examined in this study and the method of analysis and the subspace design alorithm.

2.1 The Five Bar Truss

The problem under consideration was the design of a statically indeterminant five bar truss (see Figure 1) in which the design variables were the material of each of the rod elements (aluminum or titanium thus yielding 32 or 2^5 discrete combinations) and the x-coordinate of node 2 (a continuous design variable) with the objective being design for low weight. The truss was fully constrained in both x and y directions at nodes 1 and 4 and constrained to movement in the x-direction only at node 2. By posing the problem in such a manner the region of the design space associated with each of the 32 material combinations can be displayed as a continuous functional relationship between weight and the x location of node 2. The five bar truss selected for this study was subjected to two different loading conditions. The first load case (Figure 2) involved loads applied to the structure at Node 2 and Node 3. The load at Node 2 was a 5000 pound tensile load and the load at Node 3 a 1000 pound tensile load. For load case 2 (see Figure 3) the design space was determined by 5000 pound compressive loads at both Nodes 2 and 3.

2.2 Method of Analysis

Analysis of the structure was accomplished through a fully stressed design algorithm, therefore the designs produced are not minimum weight designs, but represent a class of designs which were considered "best" for this application. The fully-stressed design procedure was based upon a simple finite element code coupled with an element resizing algorithm. In the finite element algorithm the truss elements were modeled as rod elements capable of supporting only axial loads; hence, the connections between the truss elements were considered to be pinned at all nodes and supports.

The fully-stressed design optimality criteria requires that truss elements be resized such that each member was at its allowable stress limit or was at a minimum gauge constraint. Resizing was accomplished by comparing the stress in the members as determined through the finite element analysis with the yield stress for each member. The ratio of the axial stress in the member to the yield stress (tensile and compressive yield stresses were assumed to be the same in this study) was used to determine an incremental change in the area of the member according to the relationships:

$$g_r = \left|\frac{\sigma_{xx}}{\sigma_{all}}\right|$$

$$\text{If } g_r \begin{cases} \geq 1.0 & A_{new} = A^* = \left|\dfrac{F_r}{\sigma_{all}}\right| \\ 0.9 \leq g_r < 1.0 & A_{new} = A^*(g_r)^2 \\ < 0.9 & A_{new} = A^*(g_r) \end{cases}$$

and if

$$A_{new} < A_{min}, \ A_{new} = A_{min}$$

where F_r is the force in the rod. The procedure described above is an iterative process, hence a finite element analysis was performed and the rod elements resized until the area of the truss members reached either the stress constraint or minimum gauge (0.01 square inches for this study) within a tolerance of 0.1%. The tolerance was set at 0.1% since this represented a small difference between the fully-stressed design state and the final design state. Additionally, for this method of analysis a 0.1%

tolerance in the resizing algorithm was attainable with very little computational effort.

The purpose of subspace design problem described above was to providc the data to be used to develop quantitative descriptions of design space which represented the "best" designs for a specific configuration and material system. For structures which are comprised of a single material, this fully-stressed design optimality criteria converges to the least weight design for the structure regardless of the initial design vector. However, for structures containing more than one material the fully-stressed design algorithm is known to yield final designs which are not "least weight" and which were sensitive to initial conditions. Since the purpose of the subspace design process was to provide designs for the neural network design space modeling it was determined that for this study the only requirement on the subspace design problem was that it produce a consistent family of designs. In order to obtain fully-stressed designs which were consistent throughout the design space considered for this problem, the inital component sizes were manually adjusted and a set of inital sizes identified so that all the subspace design problems started from the same set of initial component sizes and they each converged to a consistent fully stressed solution.

3 DESIGN SPACE MAPPING

In order to reduce the computational expense required to optimize design spaces, neural networks were employed as approximations to the design spaces represented by the configuration and material design variables. Because of the success of feed-forward, back propagation neural networks with a sigmoid activation function in representing structural design spaces which contain only continuous design variables[1] and design spaces which contain only discrete design variables[4], these networks were utilized for the approximation of design spaces containing both discrete and continuous design variables. The NETS computer program developed at NASA Johnson[7] was used to create the neural networks employed in this study.

Some of the issues involved in using neural networks to do design space mapping are how to configure the neural network, how much training data is required to "sufficiently" train the neural network, and how is the training data selected. These issues are not independent of one another and are discussed in the following sections as they were encountered in the course of this study.

3.1 Neural Network Configuration

One of the difficulties that arises early in the process of using neural networks in any application is determining a configuration of the network which is "optimal" for the particular problem. This issue has been addressed in some detail[8,9]. In the present study, however, it was only necessary to determine a network configuration which would yield a reasonable approximation to the design space and there did not appear to be a requirement that this be an "optimal" network. Through numerical experimentation it was found that a network with 6 neurons in the input layer, 20 neurons in the only hidden layer, and 2 neurons in the output layer (a 6-20-2 network) could provide a model for the problem under consideration which was as accurate in representing the training data as other networks considered. The 6-20-2 network was selected for this study because the time associated with training this network was less than that for networks of comparable accuracy. This configuration was selected through a study in which a series of networks was trained with 160 input/output pairs (IOPs). The IOPs are sets of data representing each design from a subspace design problem. The "I" or input in the IOP represent the material and configuration design variables and the "O" or output are the structural characterisitics used as merit functions such as weight or displacement.

These candidate designs or IOPs used for the study to select the neural network configuration were evenly distributed throughout the structural design space. This training data was developed by performing subspace optimizations for structures where the design variable associated with the location of Node 2 was uniformly spaced for each of the 32 unique material combination. This represents Node 2 locations of 7.0, 8.5, 10.0, 11.5, and 13.0 inches.

The selection of the neural network representation for the design space for this simple structural problem was simplified by the fact that the a very complete representation of the entire design space could be achieved by performing a large number of the subspace design problems. Thus a *nonparametric representation* of the design space could be presented in a graphical fashion by simply plotting this exhaustive set of individual results and then comparing against the parametric representation of the design space in the form of the neural network. In most realistic design problems this cannot be done since the subspace designs are far to "costly" to allow for this exhaustive representation of the design space.

An implication of selecting the neural network by comparing its representation against the actual form of the design space is that the uncertainty of the fit of the neural network to the design space was eliminated from this particular problem by choosing the network which provided the best representation of the problem for the given set of training data. Additionally, by choosing the best mapping to the design space, information has been utilized in this problem that is not available in a realistic design problem. These considerations were not critical in this study since the goal was to demonstrate the ability of neural networks to map design spaces which contain both discrete and continuous design variables.

3.2 Neural Network Training

Once the network configuration has been selected - however it is accomplished - it is necessary to train the network to approximate the design space. But how much training data is available or necessary given that the impetus for application of neural networks is to reduce the ^ost associated with running complex analysis methods? Also, how is training data generated such that the neural network is able to adequately represent the region in the design space in which the "optimal" design resides? These questions are still open for debate, but recursive training is an area which shows promise in the resolution of some of these issues[1]. In order to analyze the potential of neural networks and not get involved with concerns which are not central to the task at hand, the ambiguity resulting from these issues was removed from the problem by training networks with "sufficient" data to characterize the known design space. In this instance sufficiency is satisfied when the general trends of the neural network approximation to the mixed discrete/continuous design space are the same as those seen in the graphical representation from the exhaustive set of subspace designs.

Neural network representations of the design space were obtained for each of the load cases discussed above. In both cases the 6-20-2 network was employed where the inputs to the networks were the material of each of the five truss elements and the location of node 2. The outputs of the network were the weight of the structure and the displacement of node 2. Since there were only two material choices for each rod element, the material property inputs to the neural network were either 0.1 or 0.9 corresponding to material 1 or 2 for each finite element.

For problems in which there are a greater number of material choices the inputs to the neural network can be described by assigning an input node to each material choice for each rod element; therefore, for a truss consisting of 5 rods and 4 candidate materials for each element, the number of input nodes would be (4)(5) = 20. The value of the inputs could then be 0.1 if the rod is not made from the material represented by the node and 0.9 if the rod is made from the material represented by the node. This was the approach used in Reference 1 for a space structure containing 113 elements and 4 material choices for each element, thus resulting in 452 input nodes. This illustrates the neural network complexity that can result from rather simple problems if prudent selection of design variables is not exercised.

Initially, a network was trained to represent the design space for load case 1. Since training data had been obtained in order to determine the network configuration, this data was used to train the first network. Therefore, the first network considered to represent load case 1 was a network trained with data which was evenly spaced throughout each of the material combinations (160 IOPs). Additionally, a network trained with only 25% (40 IOPs) of the original data which were randomly chosen from the inital set of 160. The 40 IOP set represented a nominal value of 1.25 input/output pairs per material combination; however, some material combinations were represented by three training data sets and some were represented by no training data. Although under realistic circumstances 40 analysis runs could be quite costly, here 40 training points provided the network enough data to approximate the design space without forcing the network to simply "interpolate" between sparse data points. The second load case was analyzed in the same way as the first with the addition of a network which was trained with 40 IOPs scaled in a different manner than for load case 1 as is discussed below.

3.3 Training Data Generation

The training of a neural network requires the generation of input/output pairs. For the examples illustrated here the training data was determined through the subspace design problem discussed above. Each subspace design for a given set of materials and node 2 location provided an IOP. The training data was scaled between 0.1 and 0.9 for network training after the desired number of training points were generated (160 IOPs in this case). The need to scale the training data arose due to the nature of the sigmoid activation function. Initially, the reduced set of only 40 IOPs for network training was selected from the original set of scaled data; however, by selecting the IOPs after scaling based upon the entire range of 160 candidate designs, the manner in which the network must extrapolate from the range represented by the training data is influenced. In other words, by scaling on the entire set of 160 IOPs, the network outputs were also scaled between 0.1 and 0.9, requiring only minimal extrapolation since this represents a rather "complete" representation of the entire space. By scaling the training data after it has been selected from the 160 IOP data set, only that region of the design space which is represented by the smaller set of data is found between the 0.1 and 0.9 bounds. Inputs subsequently propagated through a network scaled as such require the network to determine outputs which may well lie outside the scaled data range. In fact, for realistic design problems in which the training data are not near the optimal solution, network extrapolation is inevitable and the ability of the neural networks to "extrapolate" to improved designs is one of their most important attributes.

The issue of when to scale training data came about in this study due to the size of the problem under investigation. Since the analysis required to obtain data for the five bar truss was very fast, it was possible to generate as much training data as desired and then try to estimate the amount which would actually be used in a more realistic problem. Typically, all of the training data that is available would be used to train the network. An exception to this is that large amounts of training exist and to teach the network with all of the data would take a long time. Consequently, a subset of the available training data would be used to represent the available design information from the entire data set.

4 DESIGN SPACE OPTIMIZATION

A problem has been posed, subspace designs indentified, and the design space approximated, so how is the "best" design determined? The attractiveness of using neural networks to approximate the design space is that the cost (and time) of objective function evaluations is significantly reduced, thus the efficiency of the optimization routine, while still important, is not an overriding concern. In this study, two concepts for optimization of systems containing both discrete and continuous design variables were explored. These methods were not streamlined to produce the most efficient algorithms possible, rather it was desired to explore the issues related to optimization of mixed discrete/continuous systems. Nonetheless, comparison of these methods with exhaustive search using the neural networks was performed, not in the interest of improved computational time, but to assess the effectiveness of the optimization concepts in determining the "optimal" solution to the neural network representations.

4.1 Exhaustive Search

In order to obtain the benchmark solution to which other solutions were compared, an exhaustive search of all material combinations was performed. This optimization was conducted using Newton's method independently for each of the 32 discrete material combinations. Since the resultant design spaces for each material combination were either monotonically increasing, thus having their extremum at a limit of the continuous design variable) or had only one local minimum, Newton's method was guaranteed to converge to the optimal "configuration" solution for each material combination. The tolerance for solution convergence was on the order of 10^{-4} inches. Then one was able to simply select the minimum weight design from the set of 32 individual designs. This approach to identifying the best design for the combined continuous/discrete problem was only possible due to the small overall design space for this sample problem - that is one of the reasons that this problem was selected.

4.2 Simulated Annealing (SA)

Generally a simulated annealing algorithm is applied to systems containing discrete variables and is effective for determining the region in the design space in which the global optimum solution exists[4]. In this study simulated annealing was explored as an optimization alternative since the inputs to the neural network were predominantly discrete (5 discrete variables and only one continuous variable). The issue was then how to handle the continuous design variable in this type of algorithm.

The principle on which simulated annealing is based is that a new candidate design is generated and accepted as the current design if the "energy" of the new design is less than or marginally greater than the previous design. In the annealing process the "energy" is related to the temperature of the material and at a given temperature the energy can assume any value in an appropriate statistical distribution. As the temperature is reduced, the allowable "excursions" in energy also are reduced. In the current application, the "energy" of the system is the weight which is a direct function of the design variables. Figure 4 presents a flowchart outlining the logic used in the simulated annealing algorithm. In this application the energy of a design is the objective function (the weight of the structure). The probability of accepting a candidate design is based on the energy (weight) of the previous design as compared to the energy (weight) of the current design and the "temperature". In the application of "simulated" annealing, the temperature is a control parameter which is used to decide whether or not "changes in energy" (i.e. weight) will be accepted as new designs. In order to apply this algorithm a "cooling schedule" must be established which controls the rate at which the "energy" or weight is reduced. Although SA algorithms are most often used in optimization problems containing discrete design variables, there is no requirement that the design vector be comprised of discrete design variables. In this study SA was implemented for a system containing both continuous and discrete design variables.

The utilization of simulated annealing in discrete optimization problems is influenced by the candidate design selection process and convergence properties. In traditional SA algorithms candidate designs are randomly determined and are entirely independent of the current "best" design. In this study potential designs were determined by perturbing the current design. This was achieved by altering a random subset of the total number of design variables (from 1 - 6 in this case). In this way an entirely new design vector was obtained approximately 17% of the time; therefore, the remaining 83% of the time design vectors which were generated contain at least some of the information from the current design. This procedure retained the ability to escape from local extrema in the design space while utilizing design space knowledge to determine new designs. This was made possible by the way in which the continuous design variable was perturbed.

The continuous design variable (Node 2 location in this case) was altered by perturbing it about the previous value with a perturbation size from ≈0.1% - 10% of the total range. The amount by which the variable was altered was "discrete" in the sense that it is fixed by the resolution of the computing system or by the choice of minimum and maximum percent change; however, by changing the continuous variable by a random percentage of itself it was possible for the variable to take on any value in the design space. This allowed the algorithm to approach the "optimal" solution without requiring the use of traditional continuous design space optimization strategies.

4.3 Successive Simulated Annealing (SSA)

The Successive Simulated Annealing (SSA) algorithm is a heuristic optimization procedure based on traditional simulated annealing techniques. The primary difference between Successive Simulated Annealing and the simulated annealing approach described above is the way in which the continuous design variable was manipulated. In the SSA procedure the continuous design variable was discretized in a rather "coarse" manner over the entire range of the continuous design variable. simulated annealing was performed using this coursely discretized design space. Based upon the best design identified using this coarse resolution of the design space a new allowable range for the continuous design variable was selected and the design space was again discretized. The resolution of the discretization in this reduced range about the preliminary solution was much finer, and simulated annealing was performed again. This procedure was continued until no change in the discrete design variables was detected between successive discretizations. The flowchart for this algorithm is presented in Figure 5.

Once this procedure converged, it was assumed that algorithm had provided the material systems which would allow for the best design. As one final step in the process a simple configuration optimization was performed to finalize the Node 2 location. An optimization again using Newton's method for a fixed set of material properties was performed with the starting value of the node 2 location obtained from the most recent SA analysis. Design generation for the SSA algorithm was performed as in the simulated annealing procedure described above except that the continuous design variable was allowed to assume only discrete values in the current allowable range.

5 RESULTS AND DISCUSSION

Since one of the primary goals of this study was to approximate design spaces using neural networks, it was useful to have the ability to graphically represent the design spaces of interest. For this study the design space consisted of a "continuous" relationship between the weight of the structure and the x-location of node 2 of the structure for each material combination. The results of performing the fully-stressed design procedure on the truss subjected to load case 1 showed a monotonically increasing (almost linear) relationship between the truss weight and node 2 x-location for all material combinations. It is due to this monotonicity that an alternative load case was considered. Even though the design space for load case 1 was relatively benign, it was still used to determine the ability of the neural network to represent this discrete design space.

A 6-20-2 neural network was trained with data from the subspace component design for load case 1. The ability of the neural network to represent this design space can be graphically illustrated by the functional dependence of weight on node 2 location. Figures 6-8 illustrate this dependence for three representative material combinations. For this load case a neural network was initially trained with 160 training data sets (IOPs) which were evenly distributed throughout the design space; each of the 32 material combinations. The notation used in this report to indicate the material system for the structure uses "1" to represent aluminum and "2" for titanium. The material system is indicated as a sequence of numbers representing the composition of truss members 1 through 5 respectively, (e.g. 12121, has element 1 made of aluminum, element 2 of titanium, etc.). The node 2 location in the training data was evenly distributed at nodal locations 7.0 inches, 8.5 inches, 10.0 inches, 11.5 inches and 13.0 inches (denoted by circles in the figures). Once the networks were developed using the training data, one could evaluate the structural weight using the neural network alone. The "lines" plotted in the results presented in the report are developed by propogating a design identified by its material combination and node 2 location (input design variables) throught the network to determine the output (structural weight).

In Figures 6-8 it can be seen that the neural network trained with 160 IOPs was able to map the design space very accurately. For this reason it was concluded that the design space could be adequately modelled with fewer training data points (and a corresponding reduction in computation cost from that required for the 160 training sets). The decision was made to represent the design space with 40 randomly selected training data points (25% of the original network training data), nominally 1.25 IOPs/material combination.

The 40 data points used to train the subsequent network were obtained from the original data set of 160 scaled IOPs. Since the original training data was scaled to the range of the inputs and outputs of the larger data set, information was being added to the design space and the neural network was required to extrapolate network outputs in a smaller range than if the data were scaled to the maximum and minimum values of the reduced data set itself. For example, in the 160 IOP data set the minimum weight in the set was scaled to 0.1. If the point corresponding to the minimum weight in the 160 IOP set was not selected as one of the 40 training points, the neural network must only extrapolate to the neighborhood of 0.1 when the inputs corresponding to the minimum weight in the 160 IOP set was propagated through the network. When the data in the 40 IOP set was scaled based upon its own range, if the minimum weight in the 160 IOP set was not included in the 40 IOP set, then the neural network must be capable of determining this weight based on its corresponding inputs. This scaled weight would be less than 0.1, and in the range in which the neural network has no information concerning the design space. Outputs which are significantly less than the training minimum value of the output (based on a percentage of the full data range) are necessarily distorted from their predicted values. Caution should be exercised in the interpretation of neural network outputs which are significantly less than 0.1 or greater than 0.9 for these types of neural networks.

Three representative material combinations for the 6-20-2 neural network trained with 40 randomly selected input/output pairs for load case 1 are depicted in Figures 6-8. For a material combination in which no training data was present (Figure 6), the neural network trained with 40 IOPs was able to fairly accurately map this portion of the design space. Additionally, for other material combinations in which training data did exist, the neural network represented the training data more accurately while approximating the remainder of the design space in an acceptable fashion. This is shown in Figure 7 where one training data point was present for the material combination (1,1,2,1,1) and the training data point was almost exactly represented while the remainder of the design space for this material combination was approximated fairly well. In Figure 8 it can be seen that the design space was even more accurately modeled when two training points were present for a particular material combination.

Having demonstrated the feasibility of using neural networks in mapping mixed discrete/continuous design spaces for this simple load case, the mapping of a more complex design space was also desired. The somewhat more complex design space was provided by fully-stressed design data for load case 2. The results for load case 2 are represented in the same way as those from load case 1. The neural network representations for the load case 2 design space are depicted in Figures 9-11 and compared with the design points used to train the networks. The networks trained with 40 IOPs were represented by only one training data point for material combination (1,1,1,1,1) as shown Figure 9. In general, the training data points were well mapped by the neural networks and for material combinations which contained only one training data point the remainder of the design space was better mapped than for a material combination which contained no training data points (as in Figure 10), but not as well mapped as a material combination containing two training data points (as in Figure 11).

As before the design space was represented by neural networks trained with 160 IOPs and 40 IOPs. In addition, a third neural network trained with the same 40 IOPs but scaled based upon the range of the reduced size set was employed. All three networks were again capable of mapping the trends of the design space with some degree of success. At training points the networks were accurate in their mapping, but all networks were unable to resolve the discontinuities in "slope" in the design space with the amount of training data provided.

Another way to view the accuracy of the neural network mappings of the design space is to compare the weight predicted by the neural networks at a particular node 2 location for all material combinations. This provides a graphic representation of the "discrete" design space and helps to illustrate some of the issues related to dealing with discrete design variables. The particular node 2 location chosen for this study was in the center of the range, 10 inches. At this location the network trained with 160 IOPs represented the fully-stressed design solution quite accurately (see Figure 12). In Figure 12, the abscissa represents the material combination. There is no "logical" to sequence the various material combinations and the "shape" of this information obviously depends upon the sequence. Visual inspection allows one to determine the least weight material system for this configuration. But visual inspection, or an exhaustive search, become impossible as the total number of combinations becomes even moderate.

Figure 13 shows the same representation for the network trained with a 40 IOP set taken from the 160 IOP scaled set. For some material combinations the weight predicted by the neural network at this location were off by as much as 6%, but the minimum value of predicted weight occured for the same material combination as the minimum weight material combination as determined by the fully-stressed analysis. This trend was repeated for the neural network trained with 40 IOPs which were scaled to the minimum and maximum values within the data set as illustrated in Figure 14.

As a final attempt to more accurately map the piecewise continuous nature of the design space a 6-20-2 network was trained with 672 input/output pairs (21 training points per material combination) which were evenly distributed throughout the continuous design space. The resultant approximation of this neural network to the design space is illustrated in Figure 15 for three material combinations. The approximation of this network to the computational solution is fairly accurate, even in the region of the slope discontinuities.

The portion of Figure 15 denoted as segment 1 represents designs in which 2 of the truss elements are at minimum gauge constraints. For all material combinations the members which were at minimum gauge along this segment of the plot were always from the set of elements 3,4,5. The discontinuities which exist between segments 1 and 2 and between segments 2 and 3 of the graph occur because the number of elements at minimum gauge changes. Along segment 2 members 3, 4, and 5 are all at minimum gauge for all material combinations. On segment 3 the fully-stressed subspace component design again yields 2 members at minimum gauge from the set of members 3, 4, and 5, but not necessarily the same members as were at minimum gauge on segment 1.

Once the design space has been determined to be "adequately" approximated, the issue which remains is then if neural networks trained with relatively little data can represent the design space accurately enough to obtain the "best" solution in the design space. To address this issue, optimization of the design space using the trained neural networks was performed.

For load case 1 the optimal solution in the design space as identified by the exhaustive design space search strategy was for material combination 2,2,1,1,1. The location of node 2 of the truss in the optimal solution was determined to be 7.0 inches which is along one of the side constraints for the continuous design variable. The results of the simulated annealing and successive simulated annealing optimization strategies on the neural networks for load case 1 yield final design points which are the same as those obtained from the exhaustive analysis. This was expected since the neural network approximations to the design space for load case 1 were very close to the finite element solution. The fact that all of the methods yielded the same solution appears to be the result of the simple "shape" of the design space and the fact that the least weight design occured along a side constraint.

One of the issues involved with the generation of neural networks to represent the design space associated with a particular analysis method is the amount of training data required such that optimization of the neural network representation of the design space leads to the region in which the "best" solution exists. For load case 2 the neural network representations which incorporated 160 and 40 input/output pairs both exhibited design spaces in which the "slope" or gradient with respect to the node 2 location was continuous rather than the piecewise continuous nature of the actual design space. The concern was that this smoothing of the design space would subsequently cause the optimization routines to select a different material combination for the optimal solution. The results of applying the optimization strategies to the neural networks trained for load case 2 are given in Table 1. From an exhaustive examination of the design space as defined through finite element analysis, the best design was determined to be for material combination 2,2,1,1,1 (same labeling convention as before) with a node 2 location of 11.12 inches. Analysis of this design point yields a structural weight of 0.248 pounds.

The first item of note in Table 1 is that for all networks and optimization routines the material combination 2,2,1,1,1 was selected as the best design. For this sample problem and load case, the smoothing of the design space by the networks was such that the optimal design state remained identifiable. Having determined the appropriate values of the discrete variables, comparisons among the individual networks can be made. In all cases the method used to identify the least weight design had was not as important as the characterization of the design space by the neural network. Each technique yielded substantially the same least weight design and the only difference between them was due to the difference in the neural networks which were used to model the design space. Since the dependence of the final design on optimization procedure had essentially been eliminated from this particular problem, assessment of the networks was based upon two criteria, how well the network represented the correct node 2 location and how well the network represented the final structural design weight.

By examining the neural network approximations to the final node 2 location as shown in Figure 16, it can be seen that the 672 IOP network provided the best representation to the design space of the four networks considered here. This was the expected result; however, comparison of the two 40 IOP networks was somewhat unexpected. The network which was trained with the 40 input/output pairs which were scaled only in the range in which the data existed (the network which was required to extrapolate to obtain the "optimal" design weight) yielded better results (6.8% error) than the network which was trained with data taken from the 160 scaled input/output pairs (10% error). It should be noted that the training data points were the same for both networks and the networks were trained to the same tolerance with the same network training criteria (global momentum, training rate, ...).

Since design decisions are based on the weight of the structure, the weights as determined by the neural networks were compared in order to assess the accuracy of the network mappings. As can be seen in Table 1, the neural network trained with 40 IOPs taken from the scaled set of 160 IOPs resulted in the value of weight which was closest to the correct weight of the structure as predicted by the neural network (0.04% error); however, this network was also the furthest from the true node 2 location for a least weight design. The other three networks resulted in final designs which were only slightly worse in terms of the predicted design weight of the structure. Because the reason for integrating neural networks into optimization routines is to provide a quick, accurate approximation to analysis methods and leave the detailed calculation of results to the complex analysis tools, one should use the network to identify important trends and not necessarily to provide "exact" representations of the design space. As long as the trends in the design space remain intact and the networks lead the designer to the region in the design space in which the optimal solution exists, the value of the objective function as determined by the neural network is of less importance.

All of the work described above was conducted using 6-20-2 neural networks where the outputs to the network were the structural weight and the displacement of node 2 under loading. However, the second output, the displacement of node 2 was not considered in the optimization procedures in this study. For this reason another neural network was trained, this time with only 1 output, structural weight. This network was trained with the same 160 input/output pairs as the 160 IOP network used in load case 2. A comparison of these two networks is shown in Figure 17. The material combinations illustrated in this figure are representative of the entire design space as before. Due to the smaller number of connections between nodes which must be modified in the 6-20-1 network, this network appears to be a better approximation of the relationship between node 2 location and weight for this problem. The implications are that the selection of both the input and output used in training the neural network can influence the accuracy of the design space representation.

6 CONCLUSIONS

The primary goal of this study was a preliminary investigation of the design of systems which contain both discrete and continuous design variables. As a means of expediting this design process the use of feed-forward, back-propagation neural networks for the storage of design information for mixed discrete/continuous design spaces was explored. It was discovered that neural networks can accurately represent these mixed design spaces in which discontinuities exist when the networks are developed using adequate amounts of training data.

In the design process it would be ideal to represent the design space using as few training data points as possible (which are typically generated through costly analysis procedures). For a simple structural problem containing both discrete and continuous design variables it was demonstrated that the character of the design space could be well represented with a relatively small amount of training data. Even though certain details were not recognized in the neural network approximation to the design spaces, general trends were preserved and those design variables which were of greatest importance could be identified.

It has also been shown that design methods are available which can be used for mixed discrete/continuous design variable problems. These methods are however limited by the discontinuous nature of the discrete design problem and by the ability to predict system characteristics in an efficient manner. The discontinuous nature of the design problem can be addressed through the use of a class of genetic optimization routines, simulated annealing. Efficient prediction of system characteristics is realized through the neural networks described here. By these approaches with feed-forward, back-propagation neural networks an efficient method for the design of mixed discrete/continuous systems can be obtained.

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Number of IOPs	Optimization Procedure	Node 2 Location	Neural Network Weight	Fully Stressed Design Weight
160	Exhaustive	10.6562	0.250739	0.249499
	SA	10.6753	0.250740	0.249456
	SSA	10.6220	0.250745	0.249578
40 (Entire Space)	Exhaustive	10.09553	0.248933	0.250715
	SA	10.13846	0.248935	0.250611
	SSA	10.01639	0.248944	0.250909
40 (Data Region)	Exhaustive	10.43837	0.252408	0.250004
	SA	10.45823	0.252409	0.249957
	SSA	10.45715	0.252409	0.249960
672	Exhaustive	10.61780	0.250410	0.249550
	SA	10.51475	0.250459	0.249784
	SSA	10.58310	0.250417	0.249628
Actual Optimum		11.119895		0.24848102

Table 1. Optimization Results for Load Case 2

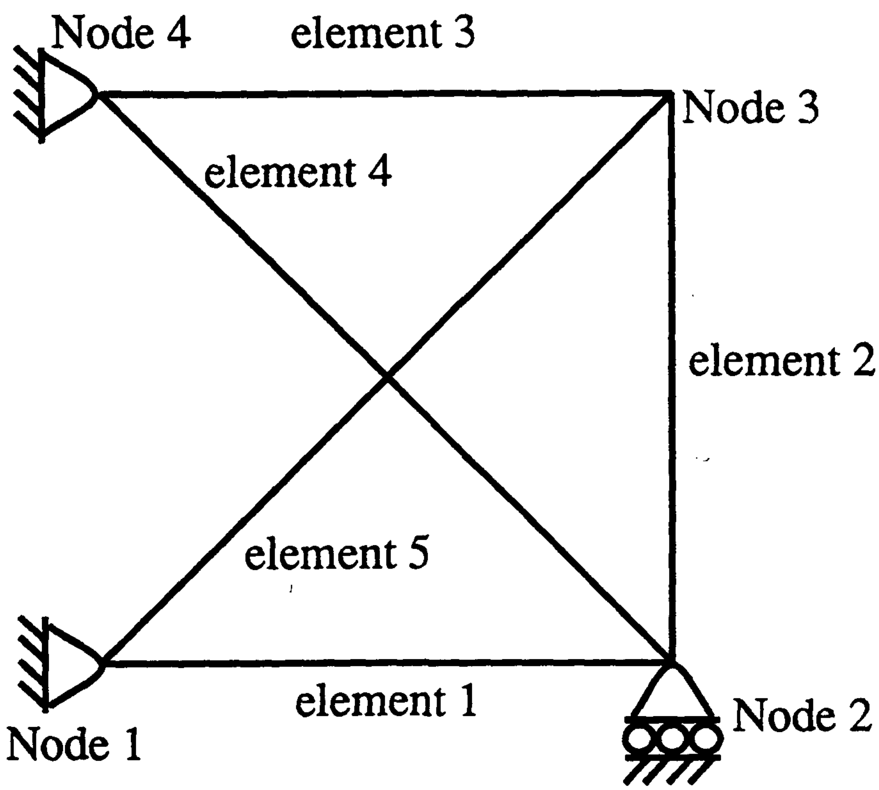

Figure 1. Truss Geometry

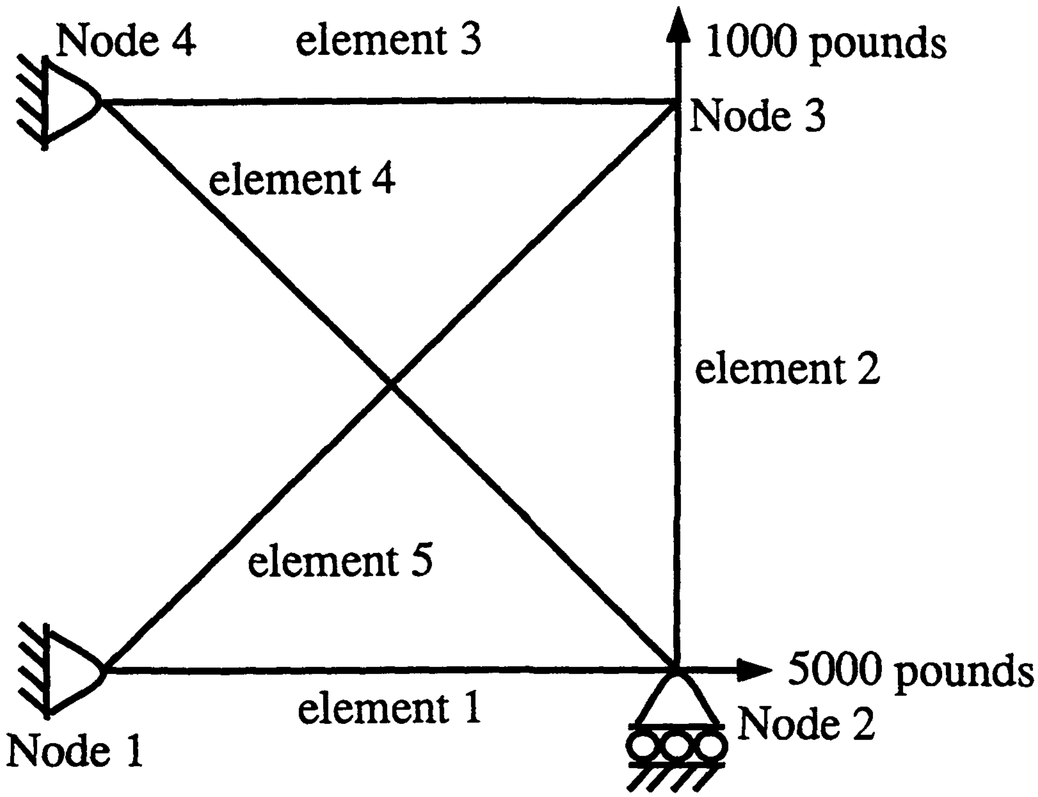

Figure 2. Truss Load Case 1

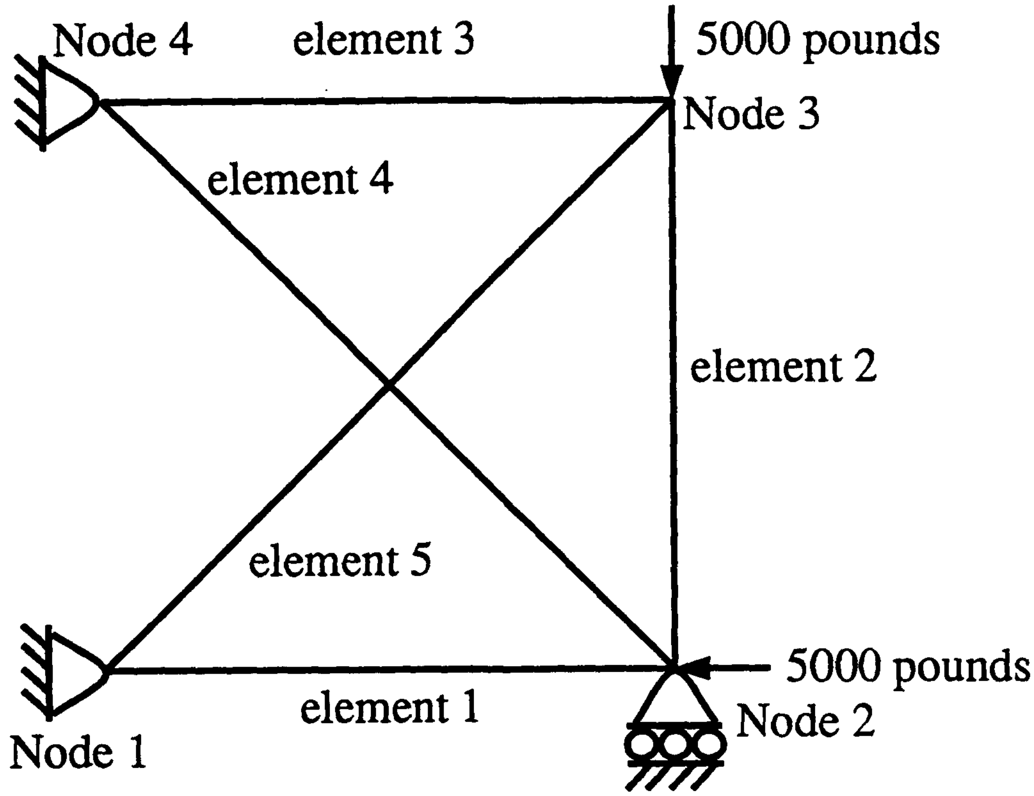

Figure 3. Truss Load Case 2

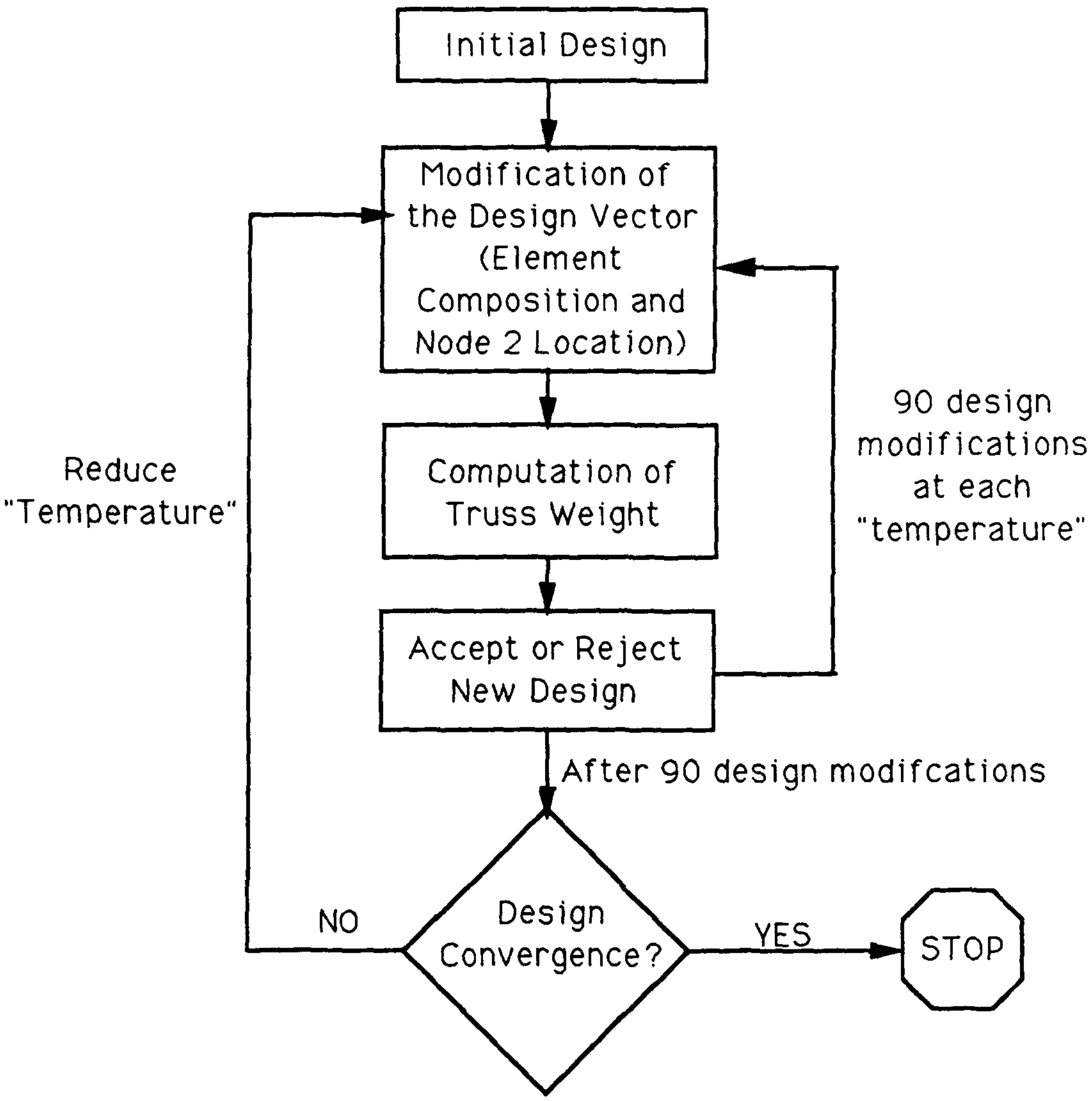

Figure 4. Schematic of Simulated Annealing Optimization Strategy

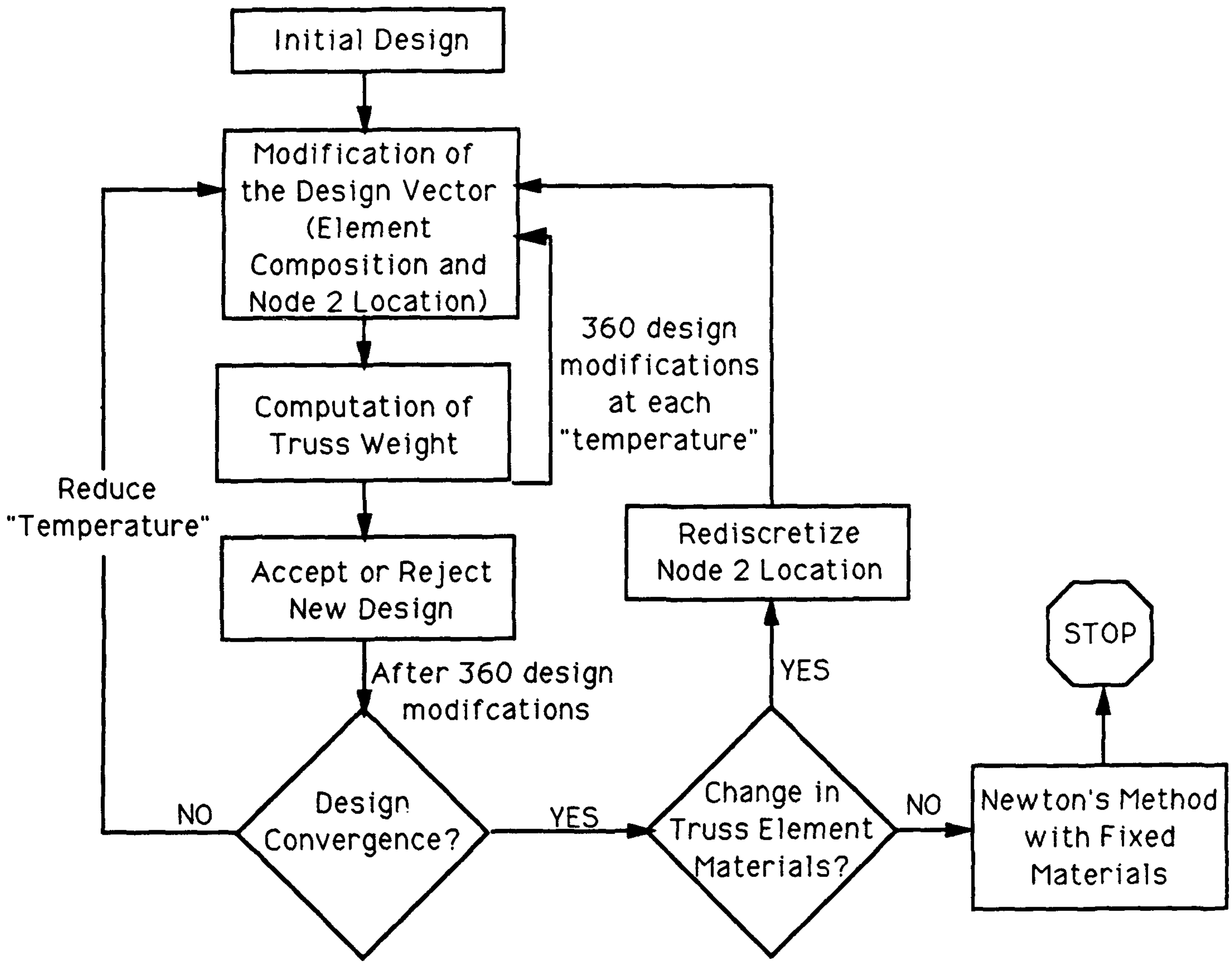

Figure 5. Schematic of Successive Simulated Annealing Strategy

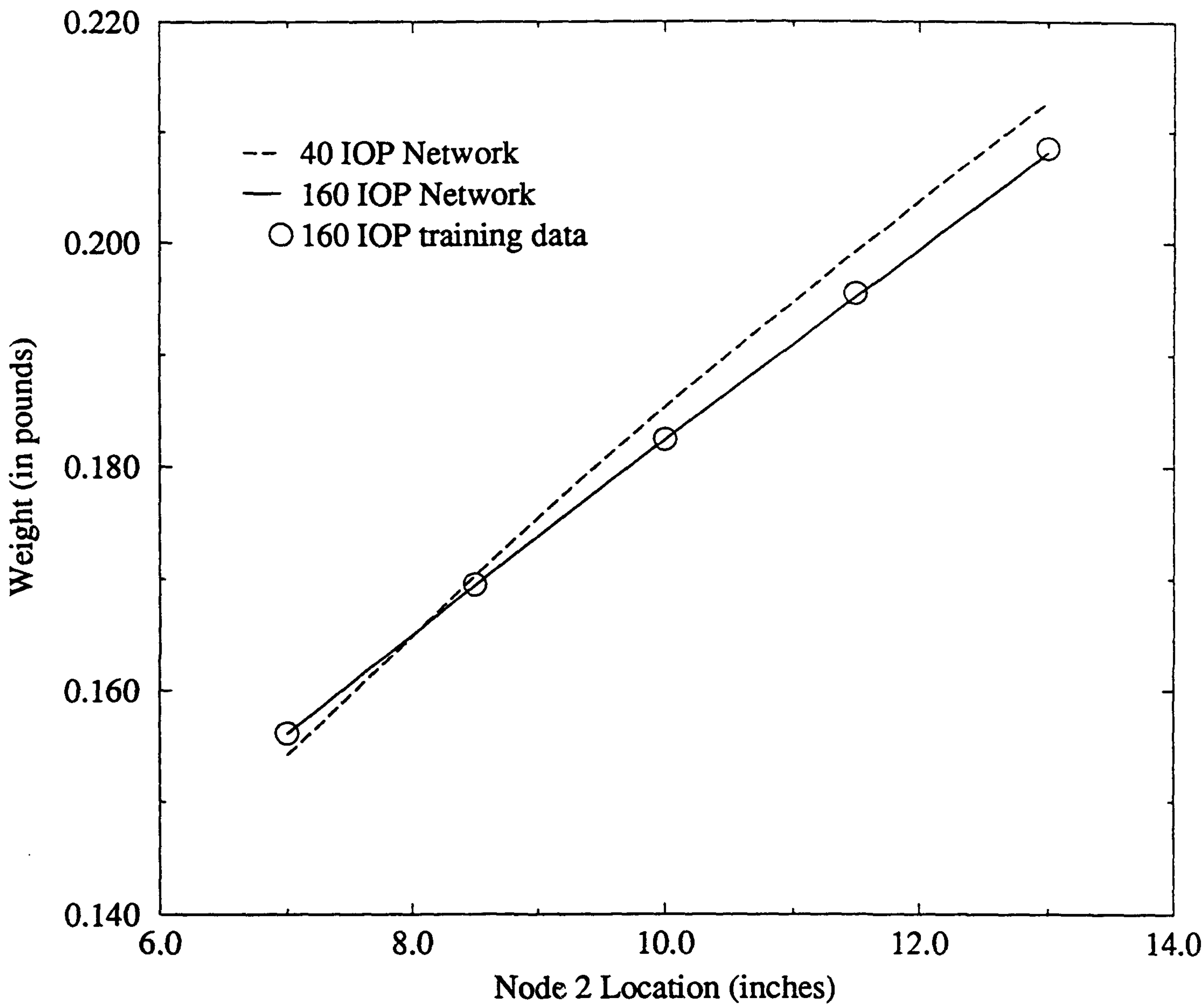

Figure 6. Neural Network Representation for Material Combination (2,1,2,2,1), Load Case 1

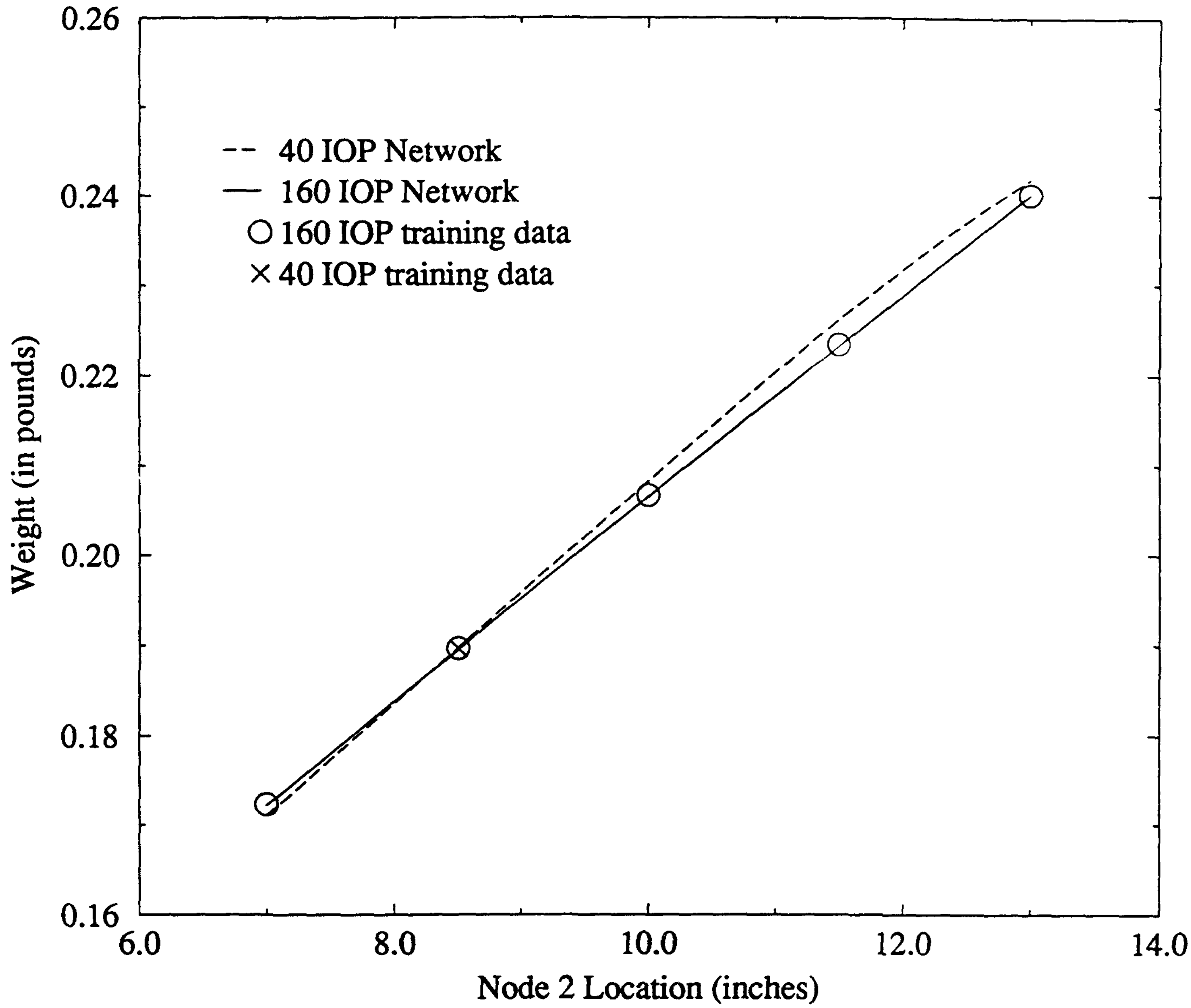

Figure 7. Neural Network Representation for Material Combination (1,1,2,1,1), Load Case 1

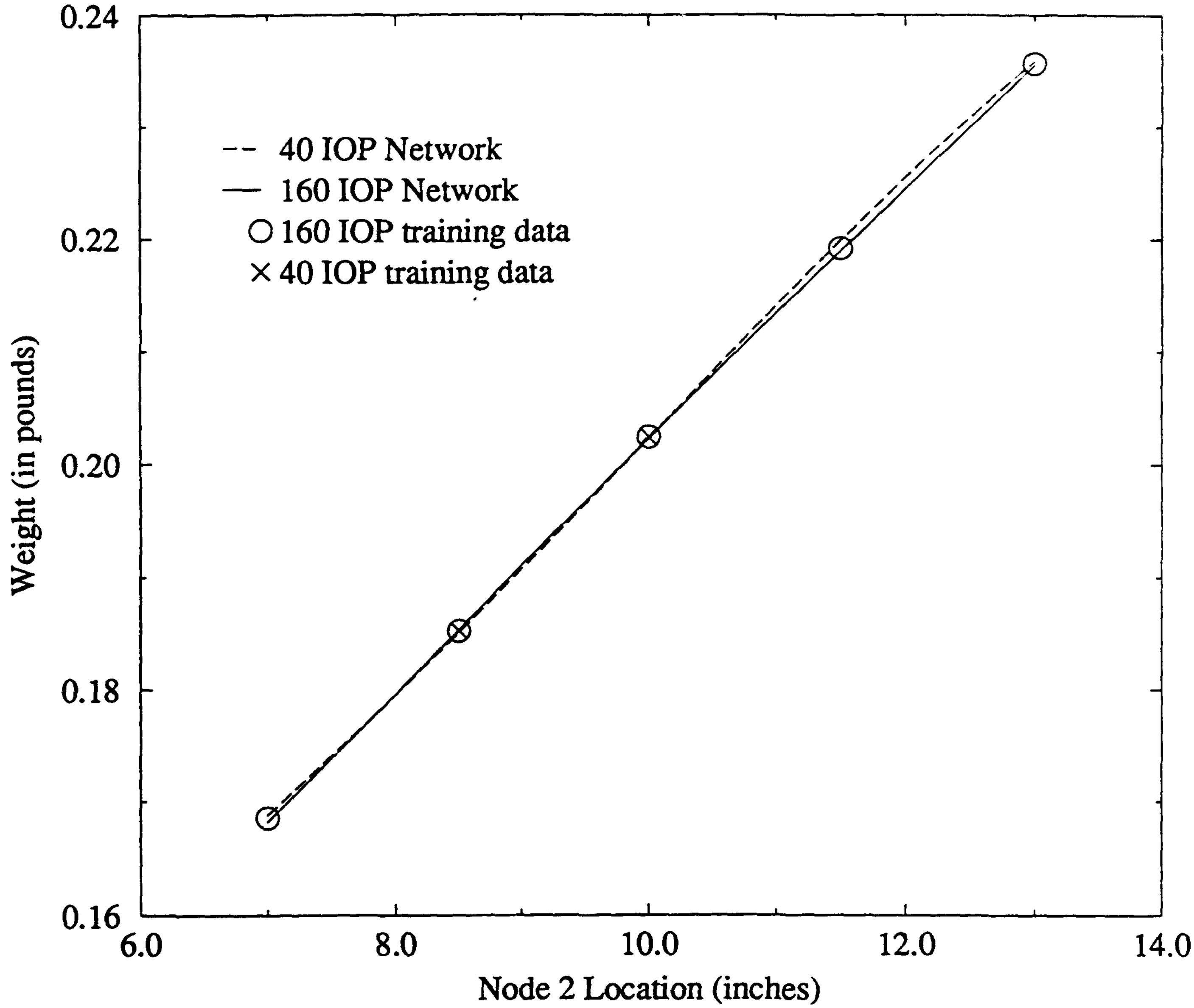

Figure 8. Neural Network Representation for Material Combination (1,2,1,1,2), Load Case 1

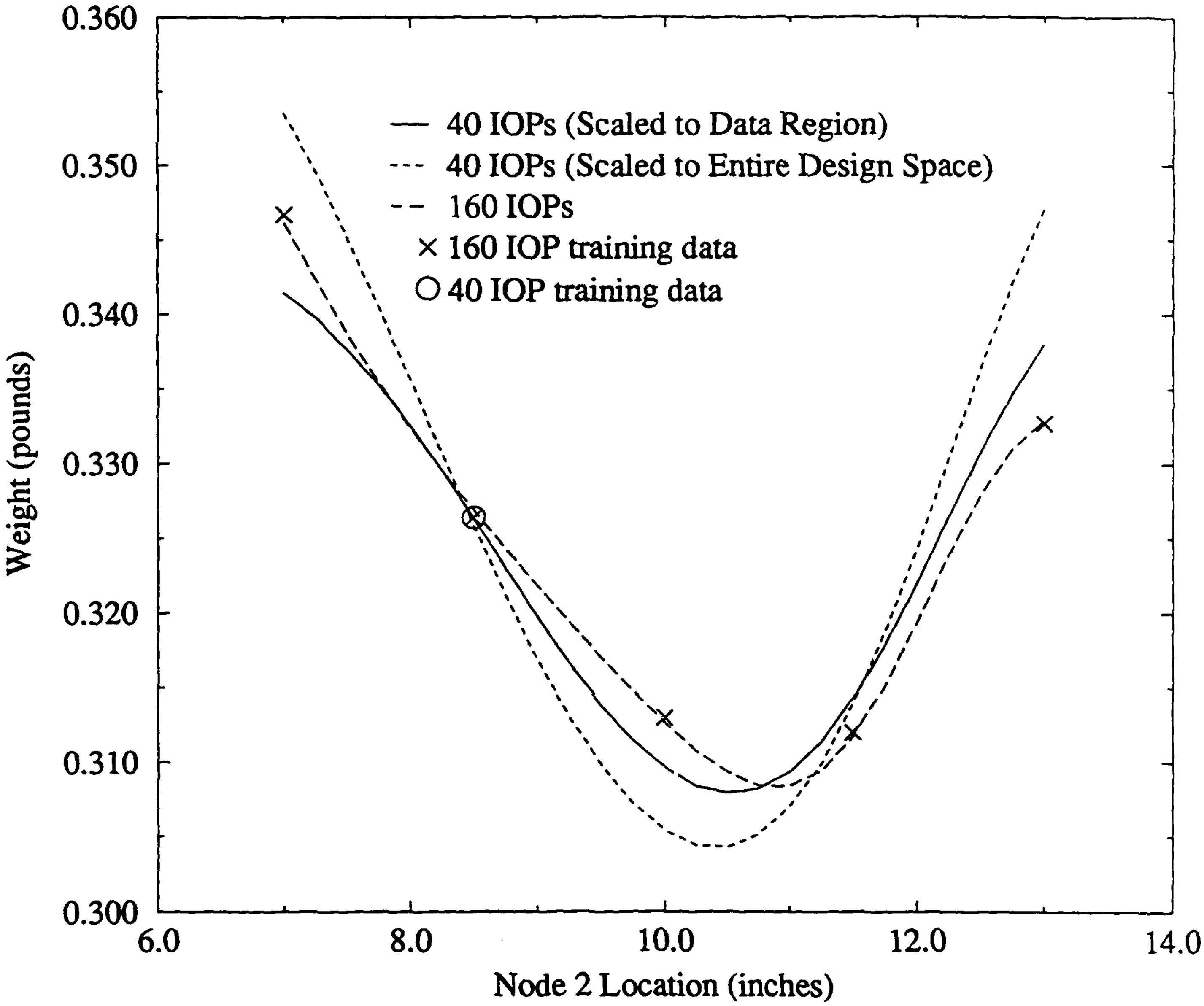

Figure 9. Neural Network Representation for Material Combination (1,1,1,1,1)

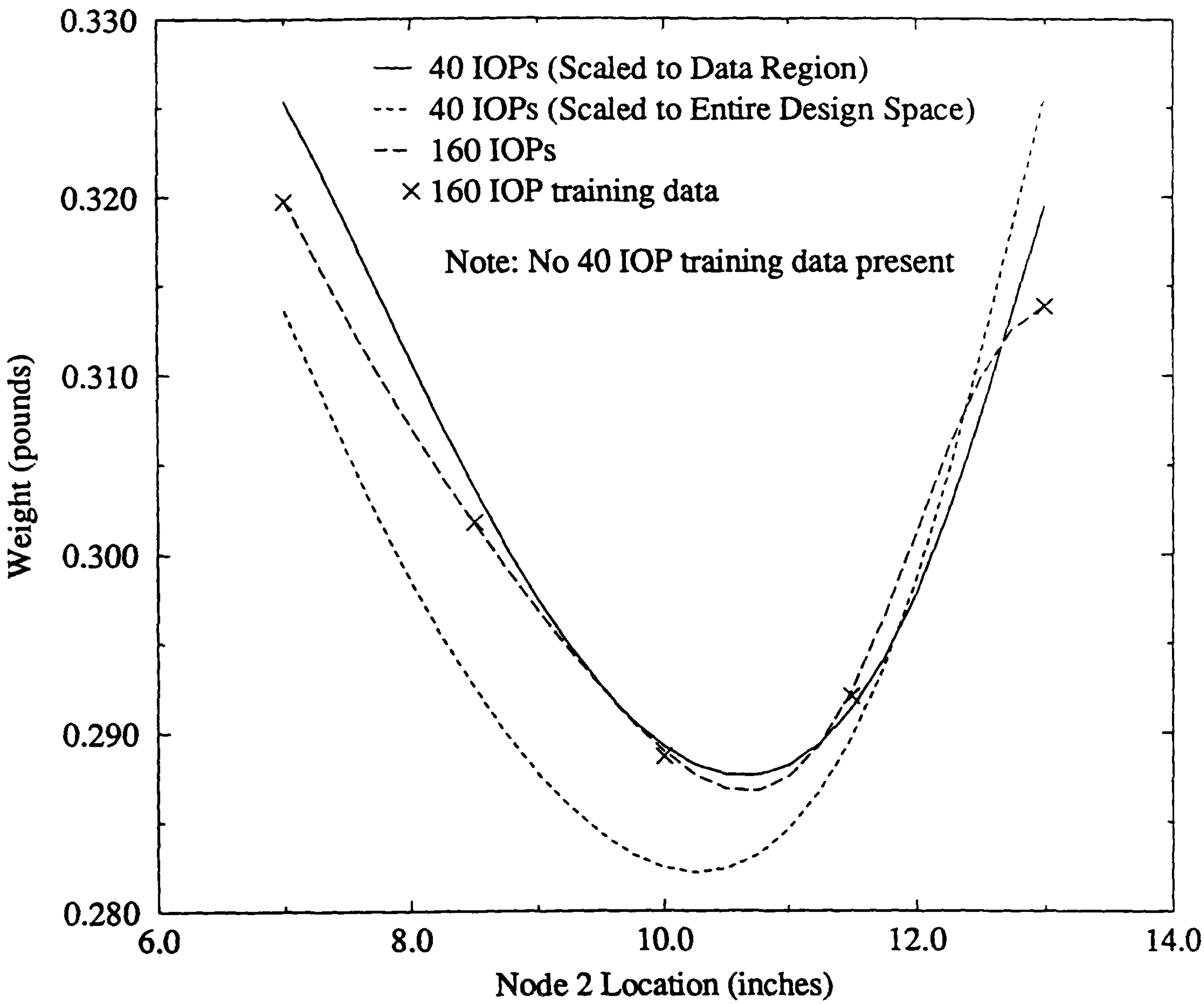

Figure 10. Neural Network Representation for Material Combination (1,2,1,2,1)

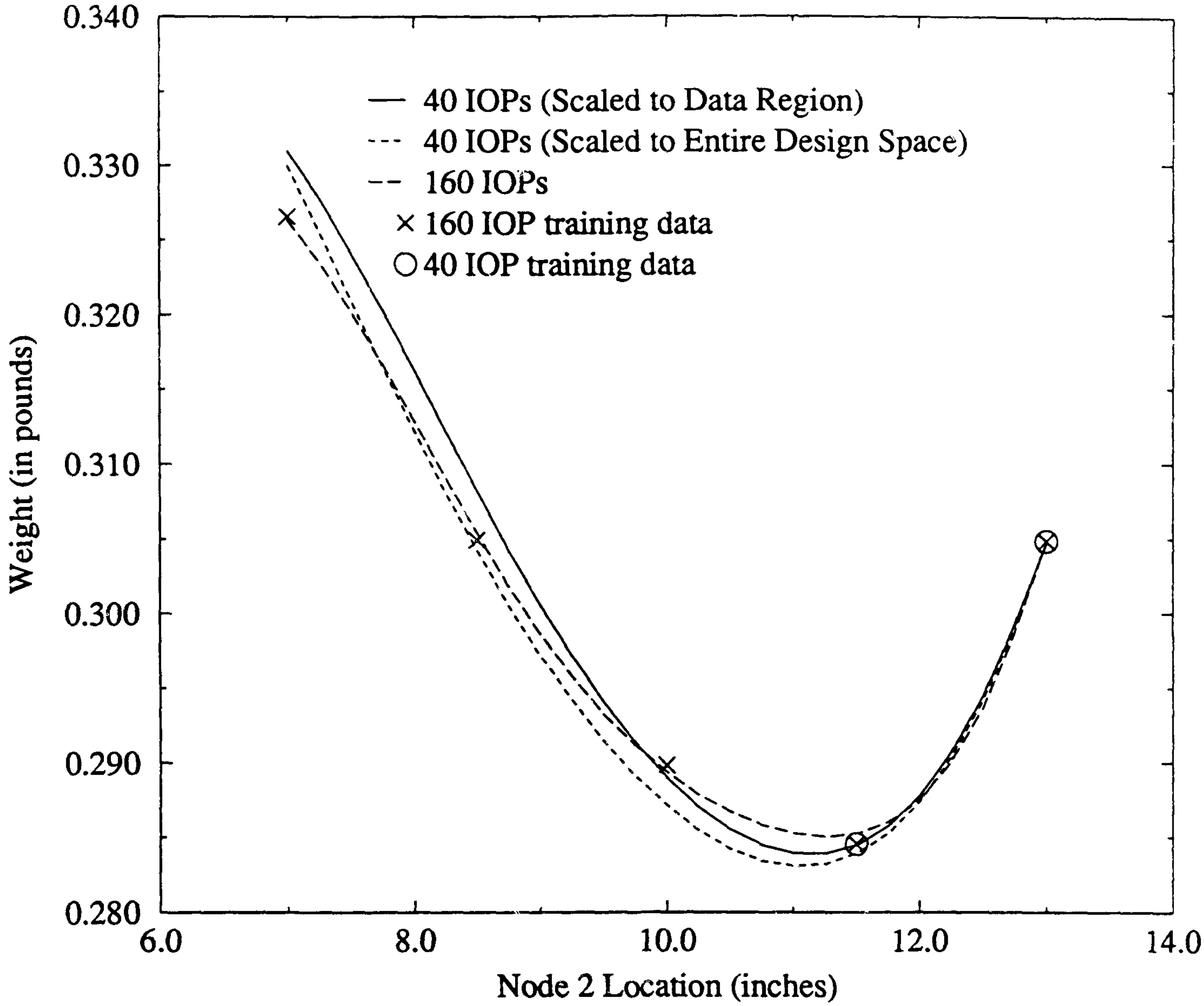

Figure 11. Neural Network Representation for Material Combination (2,1,1,1,2)

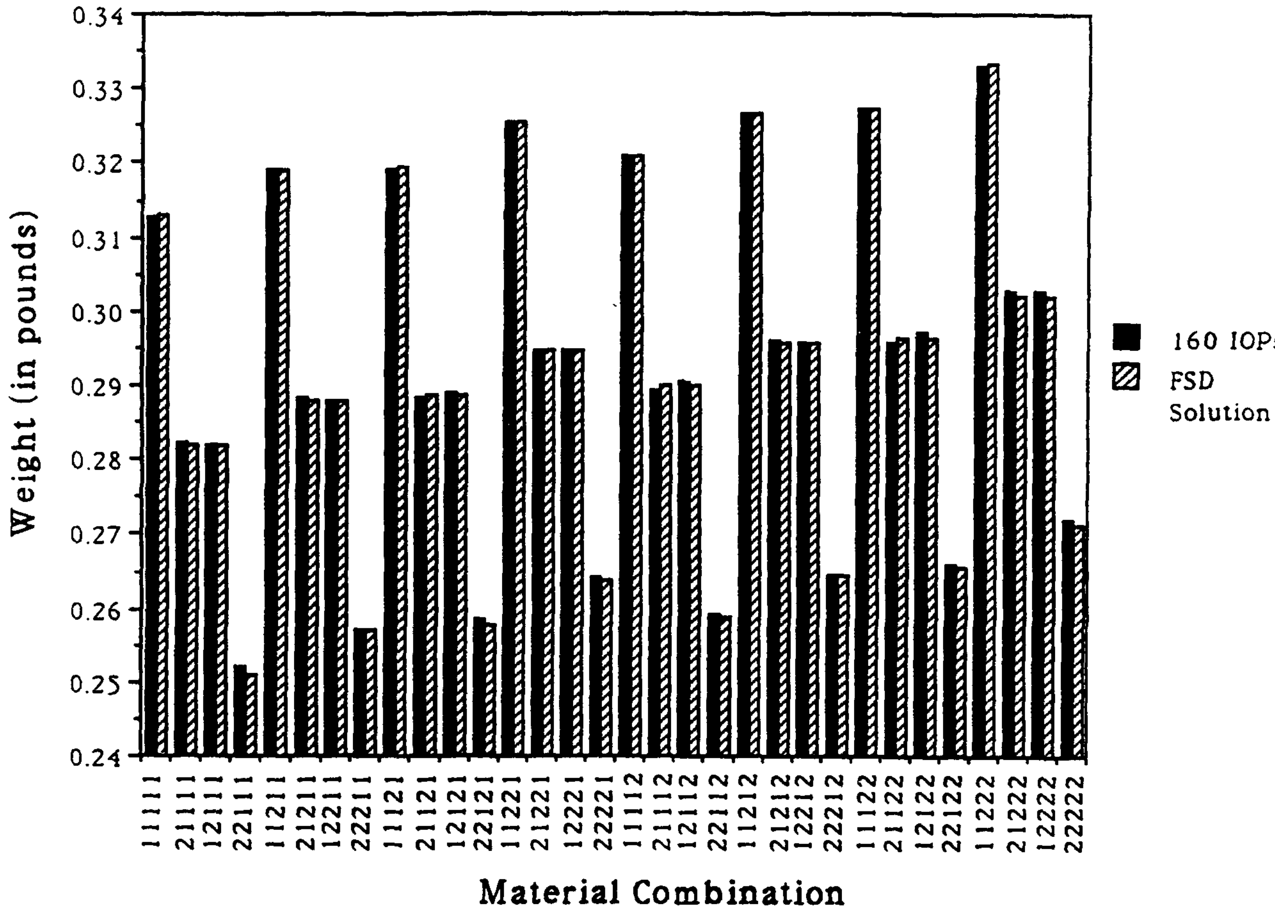

Figure 12. 160 IOP Network Representation of Structural Weight at Node 2 Location of 10 inches

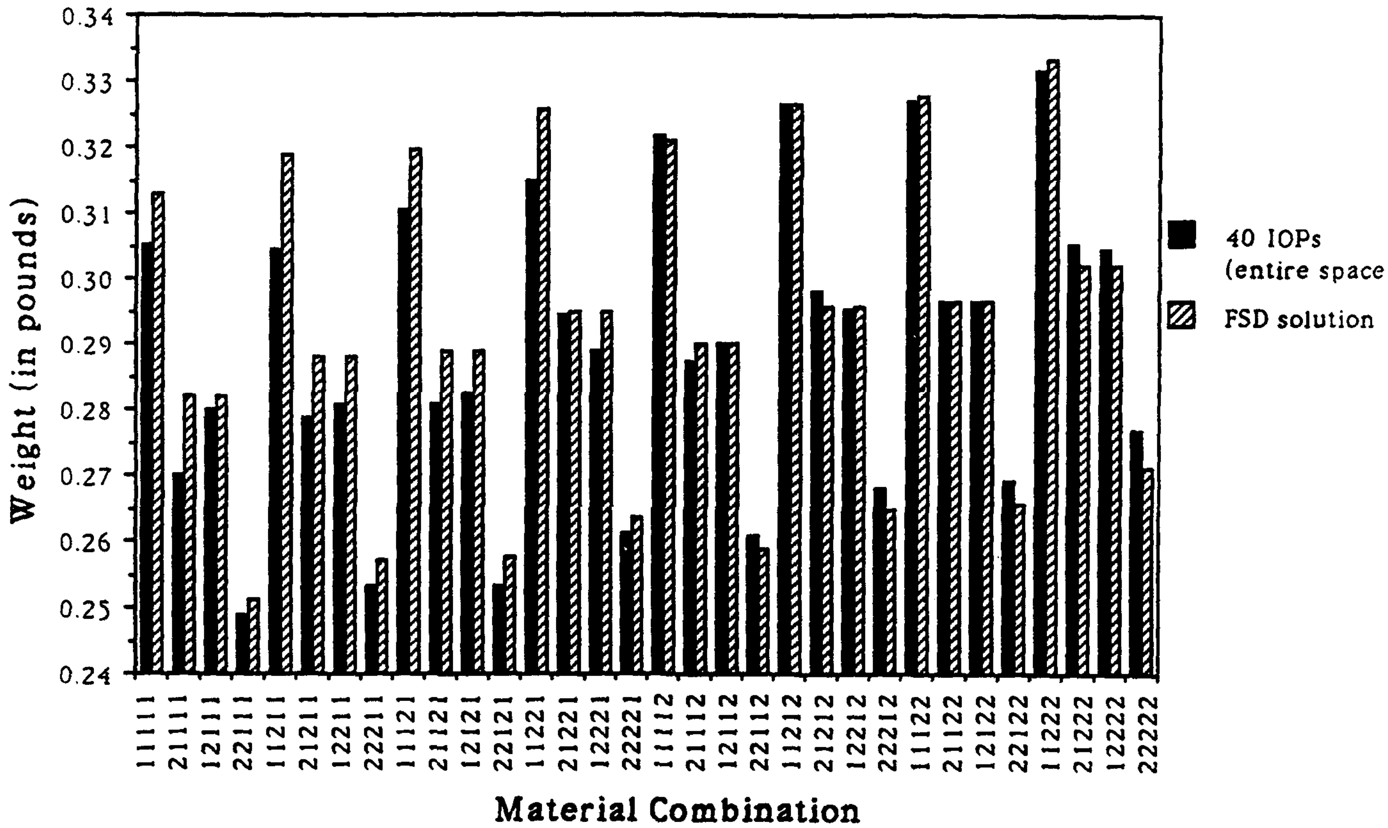

Figure 13. 40 IOP (Entire Data Set) Network Representation of Structural Weight at Node 2 Location of 10 Inches

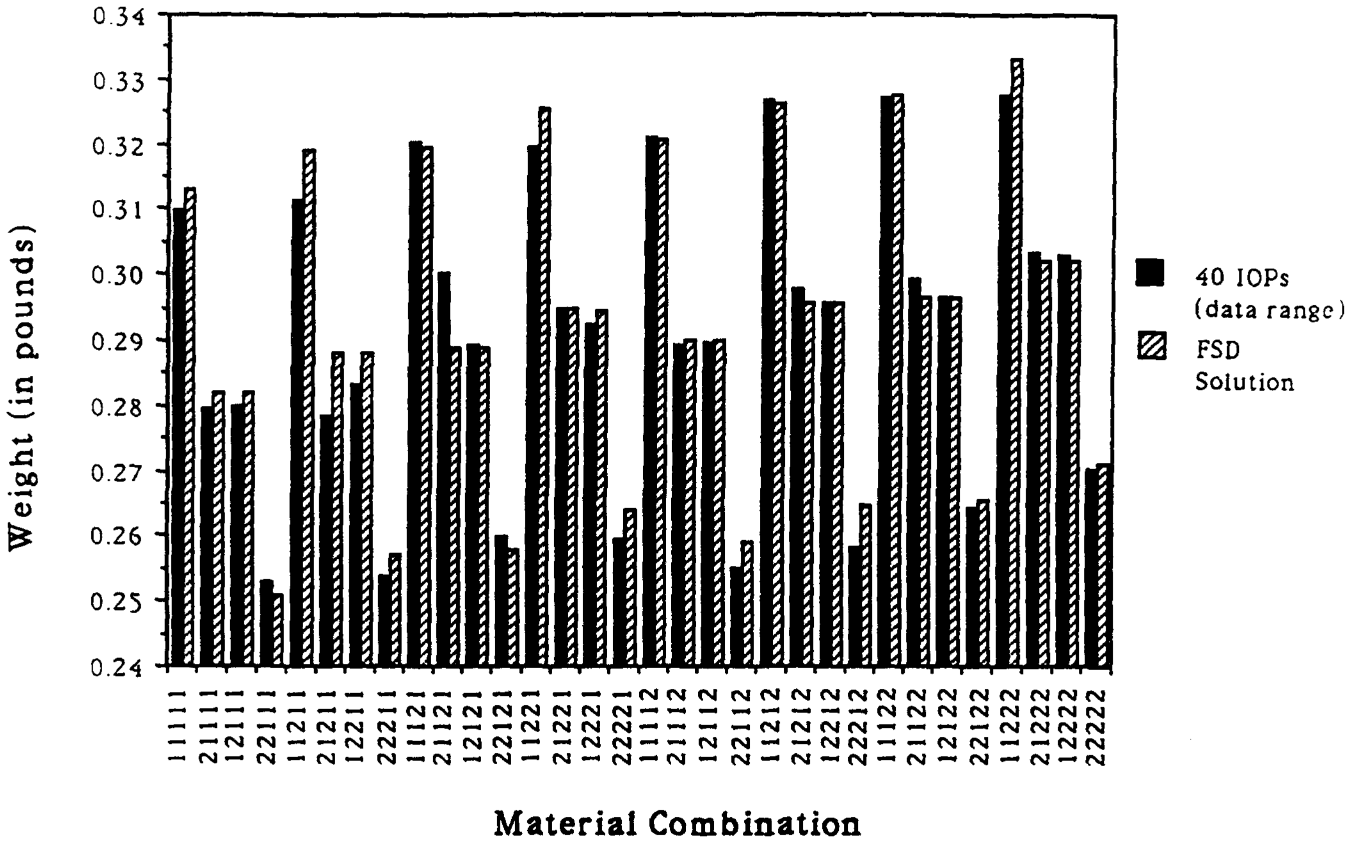

Figure 14. 40 IOP (Data Range) Network Representation of Structural Weight at Node 2 Location of 10 Inches

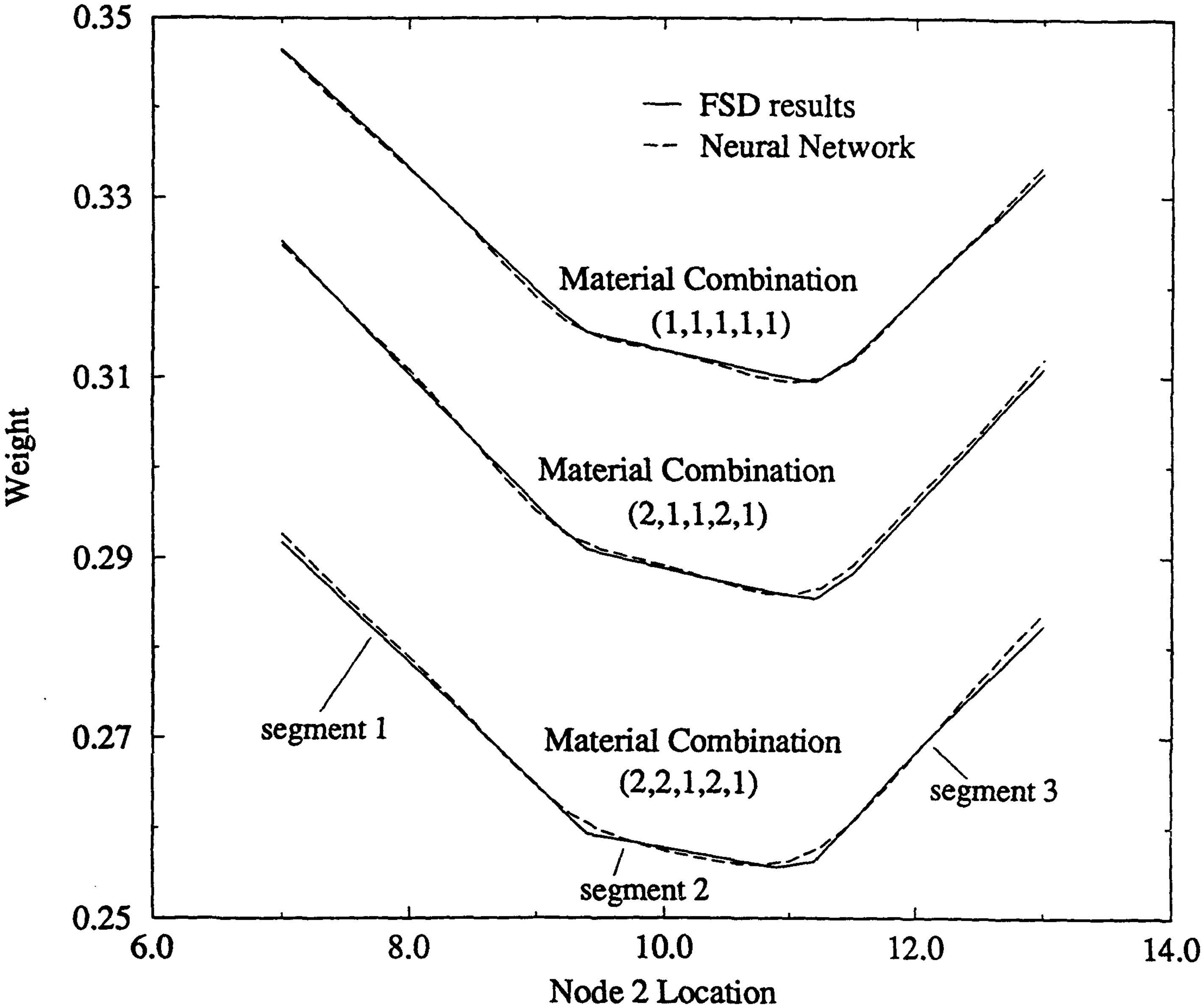

Figure 15. Ability of Neural Netorks to Map Piecewise Continuous Design Spaces: 6-20-2 network, 672 IOPs (21 IOPs/material combination)

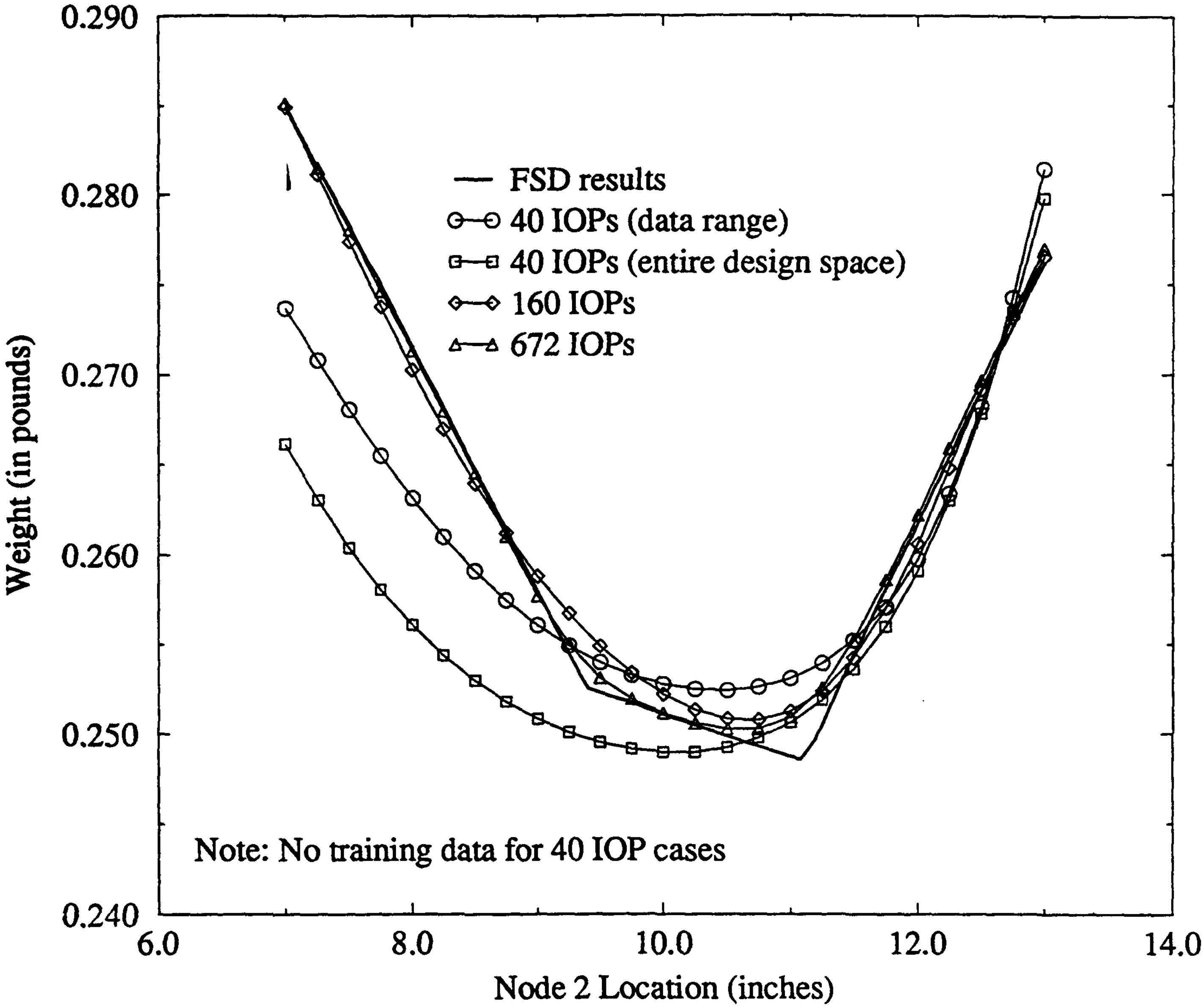

Figure 16. Neural Network Representation for Optimal Design, Material Combination (2,2,1,1,1)

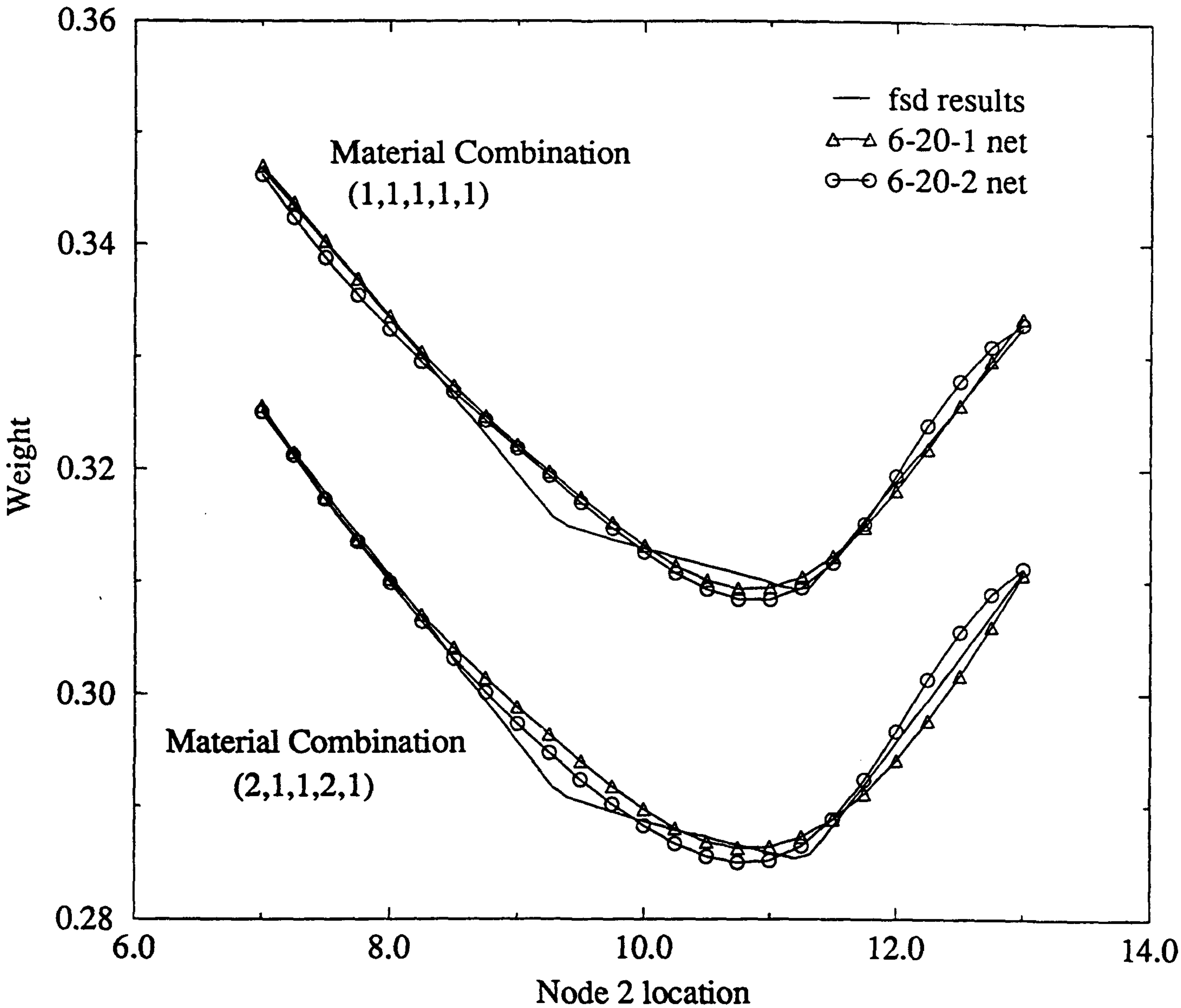

Figure 17. Comparison of Single and Multiple Output Neural Networks for Load Case 2